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STUDY OF ISOMORPHISM IN KINEMATIC CHAINS AND MECHANISMS

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Isomorphism, Mechanisms, [JJ] matrix, SCPC, MCPC

Abstract

The aim of this paper is to suggest a new and easy procedure that can be used to determine the isomorphism in kinematic chains and mechanisms. The procedure is based on theoretic approach of simple Linear Algebra theorem 'Two similar square symmetric matrices have the same characteristic polynomials'. Here a new square joint- joint [JJ] matrix is suggested whose elements are either 0 or the degrees (type) of the link. Two structural invariants namely 'sum of the absolute values of the characteristic polynomial coefficients' [SCPC] and 'maximum absolute value of the characteristic polynomial coefficient' [MCPC] of [JJ] matrices are used for mapping the kinematic chains and mechanisms. The procedure is clarified with the aid of examples. The paper is extremely useful for research scholars and designers at the conceptual stage of engineering design

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1. INTRODUCTION

Over the last several years a lot of work has been presented in the literature on the structural synthesis of kinematic chains and mechanisms. Undetected isomorphism means duplicate solutions. so, the necessity for a reliable and efficient algebraic method for this purpose is necessary. Determining isomorphism among kinematic chains using characteristic polynomials of adjacency matrices of corresponding kinematic chains are simple methods as Raicu [1], Mruthyunjaya and Raghavan [2] , Yan and Hall [3]. But the reliability of these methods was in questions as several counter examples were found by Mruthyunjaya and Balasubramanium [4]. The test proposed by [4] is based on characteristic coefficients of the 'Degree matrix' of the graph of the kinematic chains. The elements of the degree matrix were sum of the degree of vertices (degree or type of links) or unity in a link-link adjacency matrix. Later on this test was also found unreliable. Mruthyunjaya [5] proposed the representation polynomial for detecting isomorphism between two kinematic chains. The representation polynomial is the determinant of the generalized adjacency matrix, called representation matrix of the kinematic chain. But the representation matrix requires the use of a large number of symbols, the calculation and comparison of the representation polynomials is not as easy as that of the characteristic coefficients of the adjacency matrix.. One important aspect of structural synthesis is to develop the all possible arrangements of kinematic chain and their derived mechanisms for a given number of links, joints and degree of freedom, so that the designer has the liberty to select the best or optimum mechanisms according to his requirements. In the course of development of kinematic chains and mechanisms, duplication may be possible. For determining the distinct mechanisms of a kinematic chain, Several other methods like Mruthyunjaya [6] , Rao [7-13], Agrawal [14-15], Ambeker [16-17], Hasan [18-21,23], Hwang [24,27], Uicker [25], Yadav [26], Zhang [28], Yang [29], and Zou [30], etc. are available in the literature. But in most of the methods, either there is a lack of uniqueness or consume more time. Hence, it is needed to develop a computationally efficient method to determine the distinct mechanisms of a kinematic chain.

2. The Joint-Joint [JJ] Matrix

This matrix is based upon the connectivity of the joints through the links and defined, as a square symmetric matrix of size n x n, where n is the number of joints in a kinematic chain.

$$[JJ] = \left\{ L_{ij} \right\}_{n \times n} \quad \text{--------} \quad (1)$$

Where

L_{ij} { = Degree of link between i^{th} and j^{th} joints those are directly connected }

Off course all the diagonal elements L_{ii}

3. Characteristic polynomial of [JJ] matrix

D (λ) gives the characteristic polynomial of [JJ] matrix. The monic polynomial of degree n is given by equation (2).

$| (JJ - \lambda I) | = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \text{-----}\ a_{n-1}\lambda + a_n$ ---- (2)

Where; n = number of simple joints in kinematic chain and 1, a_1, a_2, a_{n-1}, a_n are the characteristic polynomial coefficients. The two important properties of the characteristic polynomials are :(a) The sum of the absolute values of the characteristic polynomial coefficients (SCPC) is an invariant for a [JJ] matrix. i.e. $| 1 | + | a_1 | + | a_2 | + \text{-----} + | a_{n-1} | + | a_n |$ = invariant and (b) The maximum absolute value of the characteristic polynomial coefficient (MCPC) is another invariant for a [JJ] matrix.

4. Isomorphism of kinematic chains

Theorem: Two similar square symmetric matrices have the same characteristic polynomials[31].

Proof: Let the two kinematic chains are represented by the two similar matrices A and B such that $B = P^{-1} AP$, taking into account that the matrix λI commutes with the matrix P and $| P^{-1} | = | P |^{-1}$. Since the determinant of the product of two square matrices equals the product of their determinants, we have: $| B - \lambda I | = | P^{-1}A P - \lambda I | = | P^{-1}(A - \lambda I) P | = | P^{-1} | | (A - \lambda I) | | P | = | A - \lambda I |$ .Hence, D (λ) of 'A' matrix = D (λ) of 'B' matrix. D (λ) =characteristic polynomial of the matrix. It means that if D (λ) of two [JJ] matrices representing two kinematic chains is same, their structural invariants 'SCPC' and 'MCPC' will also be same and the two kinematic chains are isomorphic otherwise non-isomorphic chains.

5. Procedure

The values of characteristic polynomial coefficients are invariants for a [JJ] matrix. To make these [JJ] matrix characteristic polynomial coefficients as a powerful single number characteristic index, new composite invariants have been proposed. These invariants are ‘SCPC’ and ‘MCPC’. These invariants are unique for a [JJ] matrix and may be used as identification numbers to detect the isomorphism among simple jointed kinematic chains. The characteristic polynomial coefficients values are the characteristic invariants for the kinematic chains. Many investigators have reported co-spectral graph (non-isomorphic graph having same Eigen spectrum). But these Eigen spectra (Eigen values or characteristic polynomial) have been determined from (0, 1) adjacency matrices. The proposed [JJ] matrix provides distinct set of characteristic polynomial coefficients of the kinematic chains having co-spectral graphs. Therefore, it is verified that the structural invariants ‘SCPC’ and ‘MCPC’ are capable of characterizing all kinematic chains and mechanisms uniquely. Hence, it is possible to detect isomorphism among all the given kinematic chains.

6. Illustrative example - 1

The first example concerns two kinematic chains with 12 bars, 16 joints, one degree of freedom as shown in Fig 1 .The task is to examine whether these two chains are isomorphic.

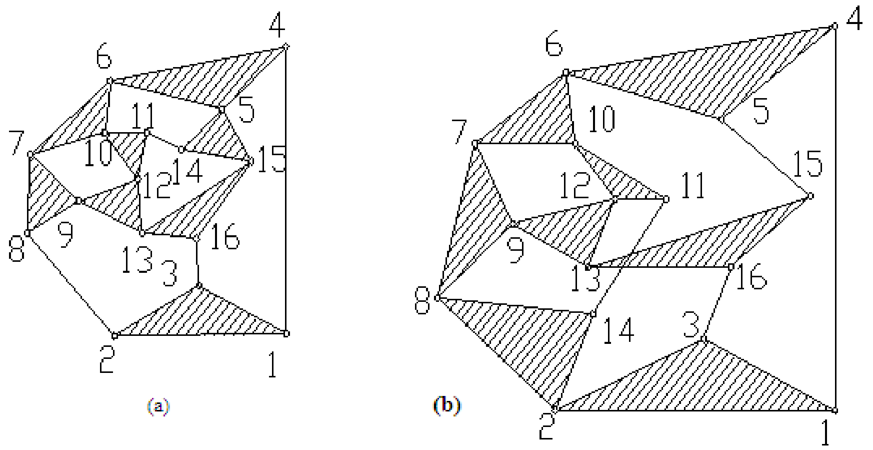

Fig 1: Twelve bar KC with single dof

[J1]and [J2] represent the [JJ] matrices for these chains respectively.The values of structural invariants are as follows:

For chain 1(a): **[**SCPC] = 2.9971e+010 , [MCPC] =1.0219e+010, For chain 1(b): **[**SCPC] = 3.2201e+010, [MCPC] =1.1287e+010.

Our method reports that kinemaic chains shown in Fig 1 (a) and Fig 1 (b) are non-isomorphic as the values of structural invariants [SCPC] and [MCPC] are different for both the kinematic chains. Note that by using another method suggested by Zhang [28], the same conclusion is obtained.

$$[J1]=\begin{pmatrix}
0&3&3&2&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&3&0&0&0&0&2&0&0&0&0&0&0&0&0\\
3&3&0&0&0&0&0&0&0&0&0&0&0&0&0&2\\
2&0&0&0&3&3&0&0&0&0&0&0&0&0&0&0\\
0&0&0&3&0&3&0&0&0&0&0&0&0&3&3&0\\
0&0&0&3&3&0&3&0&0&3&0&0&0&0&0&0\\
0&0&0&0&0&3&0&3&3&3&0&0&0&0&0&0\\
0&2&0&0&0&0&3&0&3&0&0&0&0&0&0&0\\
0&0&0&0&0&0&3&3&0&0&0&3&3&0&0&0\\
0&0&0&0&0&3&3&0&0&0&3&3&0&0&0&0\\
0&0&0&0&0&0&0&0&0&3&0&3&0&2&0&0\\
0&0&0&0&0&0&0&0&3&3&3&0&3&0&0&0\\
0&0&0&0&0&0&0&0&3&0&0&3&0&0&3&3\\
0&0&0&0&3&0&0&0&0&0&2&0&0&0&3&0\\
0&0&0&0&3&0&0&0&0&0&0&0&3&3&0&3\\
0&0&2&0&0&0&0&0&0&0&0&0&3&0&3&0
\end{pmatrix}$$

$$[J2]=\begin{pmatrix}
0&3&3&2&0&0&0&0&0&0&0&0&0&0&0&0\\
3&0&3&0&0&0&0&3&0&0&0&0&0&3&0&0\\
3&3&0&0&0&0&0&0&0&0&0&0&0&0&0&2\\
2&0&0&0&3&3&0&0&0&0&0&0&0&0&0&0\\
0&0&0&3&0&3&0&0&0&0&0&0&0&0&2&0\\
0&0&0&3&3&0&3&0&0&3&0&0&0&0&0&0\\
0&0&0&0&0&3&0&3&3&3&0&0&0&0&0&0\\
0&3&0&0&0&0&3&0&3&0&0&0&0&3&0&0\\
0&0&0&0&0&0&3&3&0&0&0&3&3&0&0&0\\
0&0&0&0&0&3&3&0&0&0&3&3&0&0&0&0\\
0&0&0&0&0&0&0&0&0&3&0&3&0&2&0&0\\
0&0&0&0&0&0&0&0&3&3&3&0&3&0&0&0\\
0&0&0&0&0&0&0&0&3&0&0&3&0&0&3&3\\
0&3&0&0&0&0&0&3&0&0&2&0&0&0&0&0\\
0&0&0&0&2&0&0&0&0&0&0&0&3&0&0&3\\
0&0&2&0&0&0&0&0&0&0&0&0&3&0&3&0
\end{pmatrix}$$

6.1. Illustrative Example-2

The non-isomorphic kinematic chains have the same characteristic polynomials using (0, 1) adjacency matrices and their kinematic graphs are called as Co-spectral graphs. But the characteristic polynomials of such chains derived from [JJ] matrices are distinct. Therefore, the structural invariants [SCPC] and [MCPC] are also distinct. This is proved with the help of examples of two kinemtic chains with 10 bars, 12 joints, three degree of freedom as shown in Fig 2 .The task is to examine whether these two chains are isomorphic.

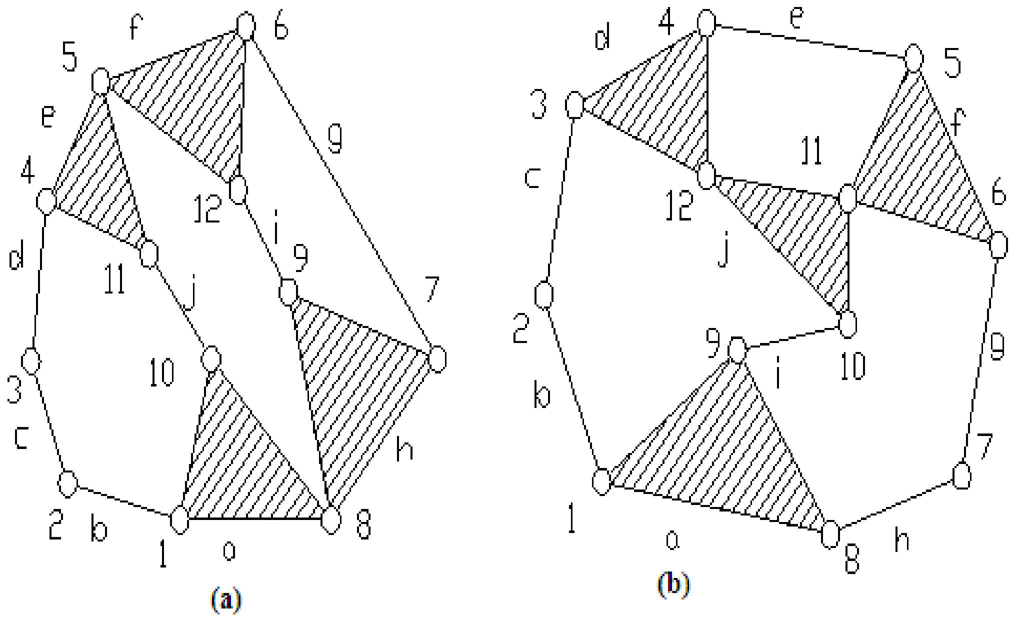

Fig 2: Ten bar KC with 3 dof

The structural invariants of these two chains are as follows:

For chain 2(a): **[**SCPC] = 8.3734e+006 , [MCPC] =3.5938e+006,For chain 2(b): **[**SCPC] = 7.0147e+006 , [MCPC] = 2.9393e+006

Our method reports that chain 2(a) and 2(b) are non-isomorphic as the set of values of [SCPC] and [MCPC] are different for both the kinematic chains. Note that by using other method comments [Cubillo and Wan, Jinbao, 2005], the same conclusion is obtained.

7. RESULTS

The proposed invariants [SCPC] and [MCPC] are used as the identification number of the kinematic chains having simple joints. The identification numbers of all 1-dof kinematic chains up to 10-Links are with Hasan [32].These invariants are also able to detect isomorphism among the kinematic chains with multiple joints also.

8. CONCLUSIONS

In this paper, a simple, efficient, and reliable method to identify isomorphism is proposed. By this method, the isomorphism of mechanisms kinematic chains can easily be identified. It incorporates all features of the kinematic chains and as such, violation of the isomorphism test is rather difficult. The method has been found to be successful in distinguishing all known 16 kinematic chain of 8-links, 230 kinematic chain of 10-links having 1-F. The advantage is that they are very easy to compute using MATLAB software.

It is not essential to determine both the structural invariants to compare two chains, only in case the [SCPC] is same then it is needed to determine [MCPC]for both kinematic chains. The [JJ] matrices can be written with very little effort, even by mere inspection of the chain. The proposed test is quite general in nature and can be used to detect isomorphism of not only planar kinematic chains of one degree of freedom, but also kinematic chains of multi degree of freedom.

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